Reg.No. ____________

Karunya UNIVERSITY

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

End Semester Examination – Nov/Dec – 2016

			Semester	**:**	**2016-17 ODD**
Code	**:**	**15MA3015**	**Duration**	**:**	**3hrs**
Sub. Name :		**Control Theory**	**Max. marks :**		**100**

ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)

Q. No.	Sub Div.	Questions	Course Outcome	Marks
1.	a.	Prove that the observed linear system $\dot{x}(t) = A(t)x,\ y(t) = H(t)x(t)$ is observable on $[0,T]$ if and only if the observability Grammian matrix $W[0,T] = \int_0^T X^*(t,0)H^*(t)H(t)X(t,0)dt$ is positive definite, where * denotes the matrix transpose.	CO1	10
	b.	Show that the second order differential equation $t^2\ddot{x} + t\dot{x} - x = 0$ with the observation $y = \dot{x}$ is observable on $[1,2]$.	CO1	10
2.	a.	State and prove a theorem for observability of non-linear system $\dot{x}(t) = A(t)x(t) + f(t, x(t)),\ y(t) = H(t)x(t)$.	CO1	20
3.	a.	Prove that the system $\dot{x}(t) = A(t)x(t) + B(t)u$ is controllable on $[0,T]$ if and only if for each vector $x_1 \in R^n$ there is a control $u \in L_m^2[0,T]$ which steers from 0 to x_1 during $[0,T]$.	CO2	8
	b.	Derive the desired control variable $u(t)$ for the control harmonic oscillator $\frac{d^2x}{dt^2} + x = u$ which steers from $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to $\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$ during the time interval $[0,T]$.	CO2	12
4.	a.	Suppose the system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$ is completely controllable and the continuous function f is bounded locally in u and satisfies the following condition (i) $\lim_{\lvert u\rvert \to 0} \frac{\lvert f(t,x,u)\rvert}{\lvert u\rvert} = 0$ uniformly in $(t,x) \in I \times R^n$ (ii) for each $r > 0$, there exists a constant L such that for every $t \in I$, $x \in R^n$, $\lvert u\rvert \le r$, we have $\lvert f(t,x,u)\rvert \le L\lvert x\rvert$. Then prove that the system $\dot{x}(t) = A(t)x(t) + B(t)u(t) + f(t, x(t), u(t))$ is completely controllable.	CO2	20

5.	a.	Let $X(t)$ be a fundamental matrix of $\dot{x}(t) = A(t)x(t)$ where $A(t)$ is a continuous $n \times n$ matrix on $J = [0, \infty)$. Then (i) Prove that the system $\dot{x}(t) = A(t)x(t)$ is stable if and only if there exists a constant $K > 0$ with $\|X(t)\| \le K, t \in J$. (ii) Prove that the system $\dot{x}(t) = A(t)x(t)$ is asymptotically stable if and only if $\|X(t)\| \to 0$ as $t \in \infty$.	CO1	12
	b.	State and prove Gronwall's inequality.		8
6.	a.	State and prove the necessary condition for a nonlinear system $\dot{x}(t) = A(t)x(t) + f(t, x(t))$ to be asymptotically stable.	CO1	10
	b.	Determine whether the solutions of the differential equations $$\frac{dx_1}{dt} = x_2 + x_1(x_1^2 + x_2^2)$$ $$\frac{dx_2}{dt} = -x_1 + x_2(x_1^2 + x_2^2)$$ are asymptotically stable.	CO1	10
7.	a.	Stabilize the system $\ddot{x} - x = u$ using the Bass method. (10)	CO1	10
	b.	Suppose that there are $m \times n$ matrices K_1, K_2 such that $A + BK_1$ and $-(A + BK_2)$ are stability matrices. Then prove that the system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$ is controllable.	CO1	10
8.	a.	Prove that the control problem $x(0) = x_0, x(T) = x_1$ for the system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$ is solvable if and only if $x_1 - e^{AT}x_0 \in C(A, B)$.	CO1	10
	b.	Verify the stabilizability of two identical mass spring system $$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \alpha \\ 0 \\ \beta \end{pmatrix} u.$$	CO1	10

9.	a.	Find the optimal control from the controllable system $\dot{x}_1(t) = x_2(t),\ \dot{x}_2(t) = u(t)$ with the cost functional $J = \frac{1}{2}\int_0^\infty \left[x_1^2(t) + 2bx_1(t)x_2(t) + ax_2^2(t) + u^2(t)\right]dt$ where we assume that $a - b^2 > 0$.	CO3	8
	b.	For the continuous nonlinear system $\dot{x}(t) = A(t)x(t) + B(t)u(t) + f(t, x(t))$ with quadratic performance criteria $J = \frac{1}{2}x^*(T)Fx(T) + \frac{1}{2}\int_0^T \left[x^*(t)Q(t)x(t) + u^*(t)R(t)u(t)\right]dt$, the optimal control exists $\lVert f(t,x) - f(t,y)\rVert \le a\lVert x - y\rVert$, where a is a positive constant and is given by $u(x(t),t) = -R^{-1}(t)B^*(t)K(t)x(t) - R^{-1}(t)B^*(t)h(t,x)$ where $K(t)$ satisfies the Riccati equation $\dot{K}(t) + K(t)A(t) + A^*(t)K(t) - K(t)S(t)K(t) + Q(t) = 0$ and $\dot{h}(t,x) = -\left[A^*(t) - K(t)B(t)R^{-1}(t)B^*(t)\right]h(t,x) - K(t)f(t,x(t)),\ h(T,x) = 0$.	CO3	12

ALL THE BEST

CO1 : Students will be able to understand the advanced concept in Control Theory
CO2 : Students are able to apply Controllability concept in their subjects
CO3 : Students are able to understand the applications of Controllability**.**